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ABSTRACT

The objective of this intervention guide is to prepare each student to be mathematically literate to survive in a world that is changing and growing as technological advances are creating new applications of geometry, measurement, statistics, and counting procedures. It accomplishes this goal by providing an intervention setting that includes manipulatives, calculators, and technology. This guide presents a list of resources for designing instructional interventions for remedial students in mathematics. An introduction to lesson planning for K-12 remedial mathematics students is also given. Contains 35 references. (ASK)

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EMPOWER! A K-12 Outcome-Based Intervention Guide for Teaching Mathematics

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EMPOWER!
A K-12 Outcome-Based Intervention Guide for Teaching Mathematics

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THE INTERVENTION GUIDE FOR TEACHING REMEDIAL MATHEMATICS

This Intervention Guide's objective is for each student to be mathematically literate — to survive in a world that is changing and growing as technological advances are creating new applications of geometry, measurement, statistics, and counting procedures. Mathematics in daily use is now much more than the shopkeeper's arithmetic of isolated skills and concepts. This intervention process in mathematics addresses the world of tomorrow, a world driven by technology and mathematics, one that will belong to the mathematically "empowered" student — the young person who can think quantitatively and solve problems qualitatively. Successful experiences in the intervention process enable the student to achieve success in reasoning, communicating, solving problems, and making mathematical connections (integration) in regular classrooms.

The intervention setting is a mathematically rich environment including manipulatives, calculators, and technology. Here the intervention teacher employs small-group or individual help for the students, providing them with the additional enriching activities, structure, and affective support that they need. These sessions give the teacher time to supplement and complement classroom instruction in assisting the individual student to construct meaning of the mathematics. The intervention teacher provides developmentally-appropriate tasks, encourages the use of models/manipulatives to illustrate the mathematical processes, and promotes student discussions about mathematics in their own words. Sessions in intervention mathematics provide students with the opportunities they need to mature mathematically and view mathematics as a worthwhile tool — a tool that everyone can use successfully to make sense out of their world and perform meaningful tasks.

The intervention process is designed to develop both the mathematical skills and the self-confidence each student needs to learn new mathematics the first time it is taught in the classroom. Thus, THE INTERVENTION PROCESS IN MATHEMATICS SHOULD LEAD THE CHILD'S DEVELOPMENT FORWARD, NOT BACKWARD — not concentrating on what the student cannot do, but empowering the child to successfully learn. In the

past, the low achieving student has been drilled to learn one isolated component of a mathematical task at a time. It is only after each sub skill has been practiced in isolation that the applications are presented to the student. By this time the student has lost interest, has been unsuccessful, and is unable to integrate the isolated skills practiced. During this time, the class itself has moved on to new concepts, thus leaving the low achiever further behind, requiring even more remedial help. This downward spiral is not the appropriate intervention process children need.

Underhill (1992) looked at mathematical remediation studies and produced a list of strategies for teachers, based on a viewpoint that a learner actively constructs one's own inner reality. Some Remediation/Intervention Recommended Strategies:

- Gather as much information as you can.
- Appeal to children's intuitive knowledge.
- Use manipulatives.
- Use pictures and diagrams.
- Use activities that promote peer interaction.
- Check progress frequently.
- Ask questions that promote reflection.
- Allow time for reflection by the student.
- Increase wait-time after questions.
- Use and encourage multiple instructional approaches.
- Use leading questions to help learners build their own meaning.
- Use mathematical recreations when appropriate.
- Look outside the classroom for intervention reinforcements.
- Relate content to out-of-school experiences.
- Stress content development and problem solving.
- Interact with learners daily, one-to-one communication.
- Conduct mini-interviews in troublesome cases.
- Be very patient.

Some Remediation/Intervention Strategies Not Recommended:

- Don't stress pencil-and-paper algorithms.
- Don't allow students to practice errors.
- Don't hurry.
- Don't jump into symbol manipulation.
- Don't overemphasize computation.
- Don't emphasize isolated, single-student activities.

IDENTIFICATION OF STUDENT OUTCOMES

The methods of identification of a student's stage of appropriate mathematical development include teacher observations, appropriate oral and written tests, interviews, student projects, and portfolios. With these, teachers compare a student's performance in mathematics with the student's performance in other subjects. Also, formal and informal observations of the student as a problem solver and as a mathematical reasoner improve the teacher's overall picture of the student's level of mathematical development. As Feuerstein (1979) proposed, the educator should gather diagnostic data on students and examine their potential for learning, rather than their deficiencies as learners (Presseisen 1988).

Intervention strategies must integrate three components: the student's cognitive style and way of learning, the mathematical concepts and skills to be taught, and the instructional procedures employed in the classroom. Intervention teachers analyze mathematical tasks and develop new approaches and teaching sequences that will "attract" the student into the learning process. The intervention goal is to substantially narrow and eventually bridge the gaps in academic performance between remedial students and their classmates. Therefore, the intervention teacher sequences student objectives into challenging and achievable steps, building the student's sense of confidence with the student's awareness of the rewards of effort. Here, both the intervention teacher and students recognize that "if a child's rate of learning is slower than that of other children, it means that the child must study harder" (Stevenson 1987, p. 8).

This intervention process identifies student learner outcomes that reflect both national and state goals for mathematics education. Nationally, one source of goals and experiences designed to empower all students mathematically is The National Council of Teachers of Mathematics (NCTM) Curriculum And Evaluation Standards for School Mathematics (1989). The educational goals of the NCTM for all students are: "(1) that they learn to value mathematics, (2) that they become confident in their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, and (5) that they learn to reason mathematically" (NCTM, 1989, p 5).

The Standards equate **knowing** mathematics with **doing** mathematics. The Standards consider how students learn mathematics, stating students:

> will accept new ideas only when their old ideas do not work or are inefficient. Furthermore, ideas are not isolated in memory but are organized and associated with the natural language that one uses and the situations one has encountered in the past. This constructive, active view of the learning process must be reflected in the way much of mathematics is taught (NCTM, 1989. p.10).

Also nationally, in Essential Mathematics for the Twenty-first Century, the National Council of Supervisors of Mathematics (NCSM, 1989) identified twelve interrelated areas of mathematics representing the required mathematical competency for all students. Essential mathematics includes: (1) problem solving, (2) communicating mathematical ideas, (3) mathematical reasoning, (4) applying mathematics to everyday situations, (5) alertness to the reasonableness of results, (6) estimation, (7) appropriate computational skills, (8) algebraic thinking, (9) measurement, (10) geometry, (11) statistics, and (12) probability.

Central to many state curriculum guidelines is the statement, "learning to solve problems is the principal reason for studying mathematics" (KDE, 1989). Furthermore, the goal of mathematics education is that ALL students develop mathematical power. All students will need to deal effectively with quantitative information. This is based on the premise that, although not all children learn in the same way on the same day, all students can learn and succeed (KDE, 1992).

DESIGNING INSTRUCTIONAL INTERVENTIONS (RESOURCES)

Effective instructional interventions for the remedial student in mathematics motivate the student to learn and individualize the instruction. "In order to motivate students to learn, teachers must both help them to appreciate the value of academic activities and make sure that they can achieve success on these activities if they apply reasonable effort" (Brophy, 1987). Hence, frequent assessment of pupil performance assists the teacher in identifying those modifications that directly address the needs of the individual student.

Since designing instructional interventions is a challenging problem solving experience, intervention teachers draw from many sources. The National Diffusion Network (NDN), a nationwide program funded by the U.S. Department of Education, is dedicated to helping local school districts and private schools improve educational opportunities and achievement for all students. Mathematics Education Programs That Work (NDN, 1991) contains reports on developed programs that have been judged to be cost efficient and proven effective. Programs in the National Diffusion Network relevant to the mathematics intervention process include:

- KinderMath, K.
- CSMP, Comprehensive School Mathematics Program, grades K-6.
- SUM, Success Understanding Mathematics, grades 2-6.
- Effective Video Disk Instruction in Core Math Concepts, grades 5-7.
- DMM, Decision-Making Math, grades 7-9.
- Sci-Math, grades 7-12.
- Sound Foundations, grades 7-12.
- STAMM, Systematic Teaching and Measuring Math, grades K-8 & 9-12.

Your state department of education's NDN contact person can assist school districts with planning workshops and inservices to incorporate these programs.

Awareness of student error patterns is a background component in the design of instructional interventions. Ashlock's Error Patterns in Computation: A Semi-Programmed Approach helps teachers work with students who make computational errors. It contains a

review of research on children's computational errors as well as guidelines for diagnosing and treating problems in elementary school mathematics. The teacher is lead to identify and analyze computational errors and to choose instructional activities to help children correct error patterns in computation. Included in the book is an extensive reference section dealing with diagnosis, remediation, and instruction in computation.

Parental involvement is another consideration in the design of instructional interventions. FAMILY MATH by the Lawrence Hall of Science is a compilation of activities to be used in workshops or at home by parents and children. It is one of the Project EQUALS' Teacher Education Programs developed at the University of California, Berkeley. FAMILY MATH focuses on parents and children learning mathematics together in their home. Designed for kindergarten through grade 8, it has been tested in inner cities, suburbs, and rural areas. Each activity page in the FAMILY MATH book includes the grade level, rationale, steps for the mathematical activity, the tool kit (a list of materials), information about the mathematics involved in the activity, and ideas for extensions. The program involves the whole family in problem solving using "hands-on" materials. Topics in the FAMILY MATH program include arithmetic, geometry, probability and statistics, measurement, estimation, calculator, computers, logical thinking, and careers. Information about organizing a FAMILY MATH workshop is included in the book.

In addition, there are many methodology textbooks available for assisting the intervention teacher. Researched methods form a basis of instructional design. For example, Teaching Mathematics in Grades K-8: Research-based Methods, edited by Thomas Post, integrates research findings with ideas and activities to improve elementary and middle school mathematics. Other useful textbooks include: Today's Mathematics (K-8) by Heddens and Speer; Elementary School Mathematics (K-8) by Van de Walle; Teaching and Learning Elementary and Middle School Mathematics (K-9) by Cruikshank and Sheffield; Teaching Secondary School Mathematics (9-12) by Posamentier and Stepelman; and Teaching Mathematics (9-12) by Sobel and Maletsky.

Many other mathematics programs and teacher resources also are available for assisting the intervention teacher. Several are described below. These are readily available from the National Council of Teachers of Mathematics Publications, Heinemann Educational Books, Lawrence Hall of Science, Dale Seymour Publications, Creative Publications, ETA, Cuisenaire, as well as local teacher resource stores. The addresses and telephone numbers of these organizations and companies are listed at the end of this paper.

Math Excursions is a three book series (Gr. K-2) of clearly organized, field-tested units for primary teachers who want an innovative extension of mathematics into art, literature, science, and social studies. Each of the five units in each book is designed to use real-world problem solving for a week or more. Black-line masters for classroom use is included.

Explorations is an activity-based primary mathematics program designed to help teachers put learning theory into practice. The program (Gr. K-2) introduces the concepts of number, geometry, and measurement with special emphasis on extensive use of manipulative material.

Math in Stride is a manipulative-based primary series (Gr. K-2) that provides children with concrete experiences and follow-up practice. It replaces drill and practice of basic skills with teaching that is interwoven with a problem solving emphasis in three broad areas: data organization, spatial relationships, and numerical relationships.

Mathematics Their Way (Gr. K-2) is a classic activity-centered program that provides more than 200 classroom activities using familiar materials. The 400 page resource guide contains over 600 photos, lesson plans, and material lists. Mathematics ...A Way of Thinking has over 300 concrete learning activities that bring Mathematics Their Way concepts to grades 3-6.

Connections: Linking Manipulatives to Mathematics is a program specifically designed to connect manipulative instruction to math topics found in most textbooks. There are 20 manipulative lessons for each grade, 1-8.

A Collection of Math Lesson Plans (Gr. 1-3; Gr. 3-6; & Gr. 6-8) is a series of 3 teacher resource books that provide practical, classroom-tested ideas to teach mathematics in

contextual situations. Included with each lesson are the purpose, rationale, activities, and samples of student works.

The Middle Grades Mathematics Project is a five-book series of activities to assist in developing problem solving and critical thinking skills. Each of the five books provides eight to ten explorations in the topics of: measurement and area, factors and multiples, similarity and equivalent fractions, spatial visualization, and probability.

Algebra Tiles (Gr. 8-12), The Algebra Lab (Gr. 5-8 & Gr. 9-12) and RecTiles (Gr. 5-9) are three different resources all using a concrete manipulative approach to develop the concept of variables and algebraic notation. Algebra Tiles and The Algebra Lab require student manipulative set materials.

The National Council of Teachers of Mathematics Curriculum and Evaluation Standards for School Mathematics Addenda Series are 16 publications to interpret and illustrate how the vision of NCTM Standards could be translated realistically into classroom practices. The six books at the K-6 levels, the five books at the 5-8 level, and the five books at the 9-12 level all have the four underlying themes of problem solving, reasoning, communication, and connections (integration) woven throughout the materials. The numerous activities in each book are all classroom-tested.

Beyond just these few listed resources are hundreds of good books on specific mathematical concepts and ideas. Please refer to the publication catalogues issued by the professional organizations and commercial companies.

THE MATHEMATICS INTERVENTION PROCESS

This mathematics intervention process for our youth has a **twofold** approach. The first objective is changing the students' affective domain — that is, improving the **student's self-image** of oneself as being successful in mathematics. Students must see themselves as capable of achieving in mathematics with guidance and effort. The second equally important objective is encouraging students to construct their own **meaning** of the mathematics. Therefore, understanding and meaning are emphasized; correct answer, drill, paper-and-pencil

algorithms are de-emphasized. Especially for the intervention/remedial student, it is the **mathematical process**, not the **product**, that is important!

This intervention guide has "world-class" expectations. It is a new, challenging direction for teachers and students. Its objective is to empower, not remediate, the student in mathematics. Thus, the mathematics examples and activities include writing, investigations, applications, projects, and non-routine problems and whose contexts vary among individual and cooperative groups, utilizing manipulatives, calculators, graphing calculators, and computers. The intervention topics covered are the same seven mathematics core concepts for the primary, middle, and high school levels. Therefore, this mathematics intervention process, although greatly different than the traditional remedial process, is closely aligned with new state assessment measures.

MODEL ACTIVITIES

The following mathematics instruction listing is based on the seven mathematics core concepts which are based upon the NCTM Standards. At three levels (primary, middle, and high school), each core concept learner outcome is described, an intervention activity lesson plan is presented, and extensions for other intervention activities are listed. Each intervention activity is designed to encourage students to discuss the mathematical experience, describing what is seen and what is done in their own words. Also, each activity allows students to connect as well as interpret many mathematical concepts and relationships as they explore each activity. These listed activities are only models. Intervention instructors will need to consult the listed resources and collaborate closely with the classroom teacher to plan daily activities.

PRIMARY SCHOOL LEVEL

1. LEARNER OUTCOME: Students demonstrate understanding of NUMBER concepts.

Number concepts are the basic building blocks of mathematics. Primary students must understand numbers, multiple representations of numbers, and counting procedures in order to understand other mathematics concepts and procedures. Students need to develop a sense of the relationships between numbers, recognizing when to perform operations on numbers, judging when the operations have been performed correctly. Learning number concepts at the primary level develops gradually and should relate to the students' experiences, use manipulatives designed to represent mathematical ideas, and always incorporate estimating when appropriate. The intervention teacher provides activities for the students that allow children to explore and to experiment with number relationships and concepts. The teacher should encourage and require the students to talk and write about their observations and conjectures.

TITLE: Spill The Beans

OBJECTIVE: Students explore families of addition (and subtraction) facts by investigating the different ways a number can be expressed as a sum of two addends.

MATERIALS: Beans, or other counters, painted one color on one side and a second color on the other side. Students can work in pairs. For each family of facts taught, each pair of students needs that number of beans, a record sheet with that number of boxes in horizontal rows, and crayons that match the two colors.

PROCEDURES: The teacher chooses the family of facts and one student in each pair takes that number of beans and spills them onto the table. That student calls out the results and the other student colors the boxes on the record sheet correspondingly. For example, 7 beans are spilled, and the call is "3 red, 4 blue" or "1 red, 6 blue." The teacher later guides the students to write the expressions 3 + 4 and 1 + 6.

EXTENSIONS: Students work in pairs to find all possible arrangements for another family of facts (for example, 9) and record their results on grid paper, organize their own lists, and discuss patterns that they observe. Students repeat these activities with different materials. Students are encouraged to repeat this activity at home.

2. *LEARNER OUTCOME: Students demonstrate understanding of concepts related to MATHEMATICAL PROCEDURES.*

Mathematical procedures enable students to find solutions to numerical questions that arise in daily life. Students apply logical thinking, estimation techniques, and proportional thinking to many problems. Students develop reasoning processes that enable them to create new procedures and products. Primary students should take risks, design their own mathematical procedure, verbalize their methods, test their conjectures, and create new rules. Their discussions reflect their understanding of the relationships between numbers and the effects of mathematical procedures on the numbers.

Intervention teachers help students connect mathematical procedures to concepts, rather than present procedures as rules to be followed. They provide their primary students with sufficient opportunities to develop a conceptual basis for a mathematical procedure before they connect it with a symbolic procedure.

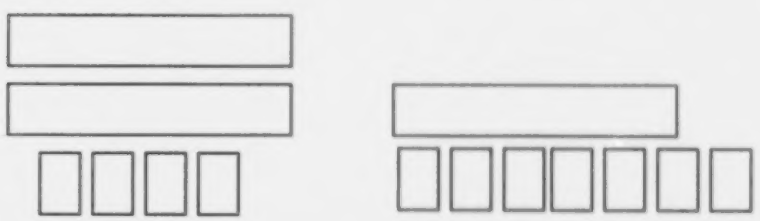
TITLE: Trading For Tens

OBJECTIVE: Students develop a procedure for adding two two-digit numbers with regrouping.

MATERIALS: Base ten blocks for each pair of students.

PROCEDURES: Students use base ten blocks to design a procedure for adding two two-digit numbers (e.g. 17 + 24). One student represents 17 with 1 rod and 7 units and the other student uses 2 rods and 4 units for 24. Their results should use the fewest blocks possible. They choose their own method to make a trade for a ten i.e., the idea of regrouping ("carrying") to represent the sum as 41, 4 rods and 1 unit. They discuss their plans and record their results.

24 (2 rods, 4 units) 17 (1 rod, 7 units)



EXTENSIONS: Additional problems lead to a discussion of when it is necessary to trade. They progress from solving the problems with models and recording their results onto solving problems without models while using the same language as if they were using the base ten blocks.

3. *LEARNER OUTCOME: Students demonstrate understanding of concepts related to SPACE and DIMENSIONALITY.*

Space and dimensionality are concepts that are used to describe the world. Students advance through stages in developing their ability to visualize and describe space and dimensionality. Van Hiele proposed that there are five levels of thought to discuss, learn, and work in geometry, namely: Level 0, students regard a figure as a whole, without considering its parts; Level 1, students analyze figures and can identify figures by their properties; Level 2, students see relationships between figures and among the properties of figures; Level 3, students begin to understand deductive reasoning; and Level 4, students have a deep understanding of mathematical structures and are able to justify statements about the structures. Students do not understand questions and assignments at higher levels than their own level. Most primary students are at the Level 1, many students still function at Level 0. Instruction should match the student's level and lead to the next thought level in geometry.

Children are familiar with three dimensional solid objects. Intervention teachers design investigations using solids to establish a foundation for the vocabulary, symbols, figures, properties, and relations of space and dimensionality.

TITLE: Solid As a Rock

OBJECTIVE: Students develop spatial and visualization skills.

MATERIALS: Rocks and other solid objects for each group of students, including a ball, cube, cone, pyramid, rectangular prism, and a cylinder (cereal boxes, cans, etc.).

PROCEDURES: Each student in the group takes two solids and tells one way they are alike and one way that they differ. Students see if each one rolls, slides, sits flat on their desk. Later, students predict what the faces (sides) of the object look like. Then students trace around the faces of the objects to get squares, rectangles, triangles, and circles. Students compare their drawings. Each group charts the number of faces, edges and corners for each solid, discussing why some solids do not have corners.

EXTENSIONS: Students build their own models with clay and toothpicks after finding real world examples.

4. *LEARNER OUTCOME: Students demonstrate understanding of MEASUREMENT concepts.*

Measurement concepts are used in daily life. Adults are often unaware of the problems caused by primary students' misconceptions about measurement. In problem solving situations, students need to select the appropriate attribute to measure, the standard unit of measurement, and the measuring tool. Students need to learn about different systems of measure, such as length, area, time, and temperature. They also need to be familiar with different standard units, such as the English system and the metric system, after using nonstandard units. To learn about measuring, students need to "do" measuring.

TITLE: 3 Shoes Equal 2 Notebooks

OBJECTIVE: Students investigate measurements of length and the need for standard units.

MATERIALS: Objects in the classroom

PROCEDURES: Students work in groups to measure the length of one dimension on 4 different objects in the classroom, such as the bottom edge of the blackboard, the height of the door, the width of their desk top, and the length of the classroom. Each group uses a different measurement device, such as a shoe, a notebook, a textbook or a pencil. They record their results and discuss how to record "parts of a shoe".

EXTENSIONS: Students graph their measurements and compare graphs. Students discuss why nonstandard units are not appropriate. They then examine metric measurements and suggest objects in the classroom they would measure with centimeters, decimeters and meters.

5. *LEARNER OUTCOME: Students demonstrate understanding of CHANGE concepts on patterns and functions.*

Change is encountered daily in the lives of students. Mathematics is used to measure and quantify change. Problem solving situations provide students with opportunities to study change and to explore patterns related to change. Working with patterns that are developed with drawings and physical objects helps primary students to develop confidence in their ability to do mathematics independently.

The intervention teacher provides a variety of activities that encourage students to measure and quantify change concepts discovered in patterns. Calculators and computers extend the patterns that students investigate.

TITLE: Calculator Patterns

OBJECTIVE: Students investigate number patterns using a calculator.

MATERIALS: Calculators.

PROCEDURES: Students enter a starting number on their calculator and then add a constant to that number repeatedly. They record the results and look for patterns. For example, start with 5 and repeatedly add 6 (use the automatic constant feature or just press the equals button on the calculator). Key in 5 + 6 = = = and record the pattern in the units place; 1,7,3,9,5,1,7,3,9,.. Start with the same number and add on a new constant, observing if the new pattern is longer or shorter before it repeats. Change the start number and examine the pattern for change.

EXTENSIONS: Have one student enter a mystery starting number and mystery constant and have a partner read the first numbers of the pattern off the calculator and then guess the mystery numbers. Change the pattern generated to multiplication by a constant.

6. *LEARNER OUTCOME: Students demonstrate understanding of concepts related to MATHEMATICAL STRUCTURE.*

Students demonstrate their knowledge of mathematical structures by relating concepts to one another and then connect (integrate) this understanding to previously learned procedures. Students use their understanding of mathematical structure to conjecture about how to solve new problems, relating the problems to procedures they encountered in other mathematical activities.

Students learn that new mathematical procedures are based on prior ones and then focus on rewriting new problems in the form that enables them to employ their previously learned procedures.

TITLE: Guess My Number

OBJECTIVE: Students investigate order relationships between numbers.

MATERIALS: Hundreds chart for each pair of students.

PROCEDURES: One player writes down a number from 1 to 100 and will only tell if the partner's guesses are too big or too small. The second player tries to guess the number in as few tries as possible. After each player has taken a turn guessing, they discuss their strategies.

EXTENSIONS: Students work together to devise what they decide is the best strategy. (Forms of this game are found on many computer programs and calculators).

7. *LEARNER OUTCOME: Students demonstrate understanding of DATA concepts related to both CERTAIN and UNCERTAIN events.*

Students live in an information society. They need to process and use data effectively. As primary students develop conceptual knowledge of probability and statistics, emphasis is placed on experimental approaches and simple plotting/graphing techniques. Intervention teachers design activities that allow the students to gather their own data; such as food preferences in the cafeteria, favorite television shows, price of candy bars, number of family members, shoe size or school attendance.

TITLE: A Picture is Worth a Thousand Words

OBJECTIVE: Students investigate graphing data they collect.

MATERIALS: Rulers

PROCEDURES: Students first choose a situation and the data they want to collect. Then they plan how to collect the data. Students discuss and record their predictions and later compare their predictions to the data they collect. They experiment with grouping the data and presenting their data with bar graphs using objects or pictures. Then students plan and produce a graph using numbers and categories. For example, students sort M&M's by color, line the M&M's up, trace around each M&M, then make a bar graph with the M&M's with colors on the horizontal and the number of M&M's on the vertical axis. Students then predict what is in a regular bag of M&M's.

EXTENSIONS: Introduce additional techniques for organizing data; such as stem and leaf plots, and computer programs that produce circle graphs. Discuss what each graph tells about the data and the advantages of the various ways of representing data.

MIDDLE SCHOOL LEVEL

1. LEARNER OUTCOME: Students demonstrate understanding of NUMBER concepts.

Students develop understanding of number concepts over time. Understanding numbers, number relationships, and counting processes are developed through connections to quantitative information encountered in everyday activities. The use of common symbols and terms helps students to understand and communicate quantities. Number is an underlying theme of mathematics. In fact, most computational and algorithmic errors at the middle school level are the results of number sense/place value misconceptions. Students explore, estimate, question, verify, and discover multiple ways to make sense of number concepts through activities that engage them in making connections rather than memorizing rules and algorithms. Calculators and manipulatives are used for investigations of number concepts. Discussions and writing about number concepts leads to further questions and explorations.

Intervention teachers choose activities that build on a student's background with number concepts and that invite the student to explore new concepts. Students develop better understanding of large numbers through experiences with problems that discuss a thousand or a million objects. Sampling and estimation techniques contribute to the making sense of large numbers.

TITLE: What does a Million Look Like?

OBJECTIVE: Students develop an understanding of large numbers.

MATERIALS: School lawn, dimensions of school lawn or dimensions of a section of the lawn, a small paper card for each student, and scissors.

PROCEDURES: Each student outlines a square centimeter on their card and then cuts out the square centimeter. They place their card on the lawn and count the number of blades of grass inside the square centimeter. Then students discuss possible strategies for finding the total number of blades of grass in the lawn, justifying their method.

EXTENSIONS: Students use measuring devices to find the dimensions of the lawn, discuss how to find the area of the lawn, and research other uses of large numbers. Encourage students to repeat this activity at home.

2. LEARNER OUTCOME: Students demonstrate understanding of concepts related to MATHEMATICAL PROCEDURES.

Understanding concepts related to mathematical procedures enables middle school students to use the procedures as tools to solve problems that involve numerical questions. Students should develop, analyze, and explain procedures for computations and techniques for estimation. They gain confidence as they select and use appropriate methods for working with mathematical procedures, employing calculators and computers in addition to paper-and-pencil methods. They develop estimation techniques to check the reasonableness of their results and to interpret the results of computations done by machines.

The intervention teacher provides activities for students that are designed to improve understanding of mathematical procedures and confidence in employing these procedures. Students enjoy engaging in mathematical recreations, and teachers can build on this natural interest.

TITLE: What Does Dinner Cost?

OBJECTIVE: Students develop estimation techniques with decimals and percents.

MATERIALS: One menu chart, with food items and prices listed, for each student and five different "dinners" (cards with food choices listed), and calculators.

PROCEDURES: Students work in pairs, and turn over one "dinner" card. They each estimate the total cost of the dinner, record the result and discuss how they estimated the total. They check their work with the calculator. Then they estimate a 15% tip, record their result, and discuss their method. They develop a way to check their results with the calculator. For each dinner, they record how close they are to the total.

EXTENSIONS: Students within 10% of the "dinners'" final price are declared "smart tippers". Thus, each student in the pair could "win". Note, teachers want to avoid competition between students for its' negative effects on self-esteem. Students pretend they have $10.00, choose their own dinner from the menu, and estimate how much change they'll receive.

3. *LEARNER OUTCOME: Students demonstrate understanding of concepts related to SPACE and DIMENSIONALITY.*

As individual students progress in their levels of understanding of space and dimensionality, conjecturing and exploring physical models are vital. Visualization of three dimensional objects and experiments with real objects form important connections for the middle school student learning to use concepts of space and dimensionality. This knowledge is later transferred into skills for the future.

The intervention teacher prepares tasks that lead students to establish relationships. Students look for patterns and generalizations, relating words and symbols to the objects they explore.

TITLE: What's "Big" For a Box?

OBJECTIVE: Students investigate volume of rectangular solids.

MATERIALS: 4 sheets of grid paper, scissors, tape, and a "filler", such as popcorn or unit cubes.

PROCEDURES: Students design the "best" cereal box with the largest volume. For the first box, students cut identical squares out of the four corners of the paper and fold up the 4 sides to make a 5 sided box. They fill the box with the "fillers" and record the size square they cut from the corners and the resulting volume. Then they pair off in groups to revise their original boxes, planning and then cutting out squares to construct boxes of largest and smallest volume.

EXTENSIONS: Discussion topics include the relationship between the dimensions of the squares and the resulting dimensions of the box with volume. Explorations progress into grouping data, charting, and then verbalizing a relationship between the number of "fillers" (volume) to the dimensions of the box.

4. LEARNER OUTCOME: Students demonstrate understanding of MEASUREMENT concepts.

Measurement concepts are used for problem solving and are a valuable tool for describing objects in the world. Measurement activities draw on the student's background and understanding of many other mathematical concepts. Intervention activities (such as ones using geoboards, polygon construction sets, or paper folding) can develop many concepts of measurement.

TITLE: The Inside and The Outside

OBJECTIVE: Students explore area and perimeter concepts.

MATERIALS: Geoboards and geobands for each group of students and grid paper.

PROCEDURES: Students use the geoboards and assign "unit square" to the area of the region enclosed by four pegs. Then, they explore on the geoboard and record on the grid paper each rectangle they find that encloses 12 unit squares (1x12, 2x6, 3x4, 4x3, 6x2, 12x1). Students design a chart to record each rectangle's dimensions, its area (12 square units) and its perimeter.

EXTENSIONS: Students look for patterns related to area and perimeter. Predict and then test rectangles that have areas that are a prime number of square units, such as 11. Compare areas of rectangles that have areas that are composite numbers, such as 15 and 24.

5. *LEARNER OUTCOME: Students demonstrate understanding of CHANGE concepts on patterns and functions.*

The study of change and patterns is fundamental to mathematics. This knowledge of change is developed. Students apply their concepts of mathematical change to other academic subjects and to daily living situations. Primary students observe patterns, then conjecture, test their conjectures, discuss their findings, and generalize the change in the patterns. As they construct understanding of the relationships found in patterns, they develop a language to describe change and compare quantities in one pattern with those in other patterns. Intervention teachers present students with problem-solving activities to explore change and to generate multiple representations of the change observed.

TITLE: Half of a Half of a Half...

OBJECTIVE: Students investigate fractional parts of a whole.

MATERIALS: Scissors and a large paper square.

PROCEDURES: The paper square represents the whole unit, 1. Students fold the square in half by matching opposite sides, and cut it into two halves. Write 1/2 on one section. Fold the other section in half and cut on the fold line. Continue folding, cutting, and marking 1/2, 1/4, 1/8, 1/16... Students predict the 5th fold, 6th fold, etc. Calculators are used to extend the pattern. They rebuild the whole square with their piece and make a chart to compare the fractional part removed to the fractional part left (1/2 to 1/2, 1/4 to 3/4, 1/8 to 7/8...). Students generate representations of fractions found, e.g. 1/8 is 1/2 of 1/4 or 1/2 of 1/2 of 1/2.

EXTENSIONS: Students use their calculators to represent the fractional parts as decimals. Older students explore moves on a number line. They start at 0 and each move takes them half the distance from their last move toward 1.

0	1/2			1
	1/2	3/4		1
		3/4	7/8	1

Students discuss the tenth move's location, which move passes 5/6, and whether the moves ever reach 1. Students graph the results and generate an algebraic representation for the pattern, e.g. (1/2)n where n is the number of folds.

6. *LEARNER OUTCOME: Students demonstrate understanding of concepts related to MATHEMATICAL STRUCTURE.*

Students who develop the ability to relate mathematical concepts to one another and then relate their understanding to previously learned procedures are "empowered" mathematically. It may seem an efficient use of time to demonstrate the structure of new concepts and procedures for students. However, giving the students the structure concepts inhibits their development of methods that bridge the gaps between their prior learning and new material, as well as their confidence in their ability to confront new situations mathematically.

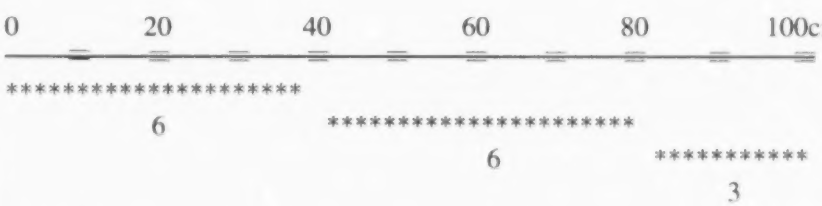
It is much more meaningful for the intervention teacher to choose activities that encourage students to develop their concepts of mathematical structure. Students need time to share their assumptions and to relate new problem situations to ones that the students have previously encountered successfully.

TITLE: A Percent of What?

OBJECTIVE: Students investigate percentage problems.

MATERIALS: Meter sticks marked in centimeters (100 parts).

PROCEDURES: The meter stick represents 100% of the missing number. The students use the meter stick to find the missing number in the problem, "$6 is 40% of some amount, what is the total amount?" Beginning discussion in groups includes predictions about the missing number, is it greater than $6, than $12, etc.? Students work together to mark 40 centimeters to represent $6, add 40 more centimeters to represent $6 more, and add the remaining 20 centimeters which represent $3, half of $6. The total amount is $(6+6+3), or $15. Some groups may need to start with problems based on using the meter stick to represent 10% or 50% of the missing amount.



EXTENSIONS: Students use base ten flats (areas of 10 x 10) to model percentage problems.

7. *LEARNER OUTCOME: Students demonstrate understanding of DATA concepts related to both CERTAIN and UNCERTAIN events.*

Students constantly are faced with certain and uncertain events in their lives. Experimental probability activities help middle school students make predictions, gather information, and then modify their predictions. This development of higher-order thinking skills benefit them as better informed consumers and citizens. Experimental probability activities also connect many mathematical procedures. Thus, the intervention teacher introduces students to probability by choosing activities that allow the students to guess about outcomes, perform experiments, and then modify and refine their original predictions.

TITLE: Pop Can Data

OBJECTIVE: Students sample a population and analyze the data.

MATERIALS: Student-designed data gathering sheets.

PROCEDURES: Students determine and implement a strategy to sample the population of the school with respect to soda pop purchases before and after school. They design a recording procedure to match their questions, such as: which class spends the most on soda pop, what brand of soda pop do boys and girls buy, what is the average spent on soda pop by each class. Students collect data for a week, chart their data and publish their results, comparing them to their predictions.

EXTENSIONS: Students revise their sampling process. Discuss the concepts of mean, median and mode. What is the "best" soda pop to have for profit? Students collect and discuss statistical information found in the newspaper. Investigate marketing and advertising.

SECONDARY SCHOOL LEVEL

The variety of classes and needs of the secondary student present the intervention teacher with many challenges. Just help with homework is itself a monumental task. Here, peer tutoring and computer assisted instruction can expand the intervention process. Beyond homework, the following activities integrate the traditional courses of algebra, geometry, and general mathematics that can be used with a wide diversity of students.

1. LEARNER OUTCOME: Students demonstrate understanding of NUMBER concepts.

Number concepts are the basic elements of mathematics. Secondary students' understanding and use of number concepts is fundamental to their mathematics studies and to their own lives in society. Their lack of mastery of number skills and lack of confidence in applying number concepts often excludes them from participating in problem solving situations. Drill and practice on what students misunderstand does not contribute to their understanding the underlying concepts. Problem solving situations form the basis for learning new concepts. Students who engage in activities using concrete materials to interpret algebraic expressions or geometric relationships such as algebra tiles, paper folding, and straw models reevaluate and reconstruct their own understanding of number concepts.

Intervention teachers present activities to connect new learning to number concepts that students understand as well as to number concepts that students do not understand completely. A real world example is the computer. Students live in a technological world and computers operate on binary language. Students research base 2 numbers and discuss why computers use the binary system (e.g., electrical current in a chip can only be off (0) or on (1)).

TITLE: How Do Computers Count?

OBJECTIVE: Students develop an understanding of the binary system.

MATERIALS: Each pair of students needs a sheet of grid paper, scissors, a large sheet of paper, and 7 two-sided markers (one side to represent "on", the other side for "off").

PROCEDURES: Students cut their grid paper into rectangles that represent powers of 2; 1(1x1), 2(2x1), 4(2x2), 8(2x4), 16(4x4), 32(4x8), 64(8x8). They then line the rectangles up horizontally, so that from right to left, the powers of 2 increase (64 is on the left, 1 is on the right). Students place the markers under the proper rectangles to represent the numbers. Thus 67 is

64	32	16	8	4	2	1	
on	off	off	off	off	on	on	
1	0	0	0	0	1	1	(base 2)

EXTENSIONS: Replace the rectangles with powers of two for place value and have students write their social security number in binary form. Challenge the students to devise a procedure to add two binary numbers on their "computer".

2. *LEARNER OUTCOME: Students demonstrate understanding of concepts related to MATHEMATICAL PROCEDURES.*

Mathematical procedures are the tools that enable students to solve quantitative questions. Technology is changing the methods available to the students to employ mathematical procedures. Students develop creative ways of transferring their knowledge of mathematical procedures and logical knowledge into effective problem solving techniques.

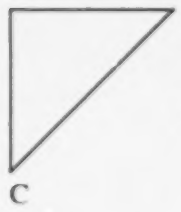
The intervention teacher provides students with activities that challenge them to blend mathematical procedures, logical thinking, and estimation techniques. Students are encouraged to consider alternative procedures and discuss the procedures' merits. Matrices are useful problem solving tools secondary school students use in problem solving that have connections with other mathematical procedures and computer uses.

TITLE: It's On The Map

OBJECTIVE: Students develop procedures to relate geometry and algebra to solve problems.

MATERIALS: A state map, ruler, compass, and calculator for each group.

PROCEDURES: Students work in small groups with the state map to find the shortest distances between three cities (A, B, C in the diagram). They measure the distances between cities with a ruler, read the scale on the map, and develop a procedure for calculating the distance between the cities. Then they construct a distance matrix (see below). The distance matrix, below right, has a variable for the distance between cities C and A. Students use the ruler and compass to construct maps corresponding to this matrix, finding largest and smallest possible distances for x.



	from A	from B	from C
to A	0	15	20
to B	15	0	25
to C	20	25	0

	from A	from B	from C
to A	0	15	x
to B	15	0	25
to C	x	25	0

EXTENSIONS: A demonstration of scaler multiplication on the computer shows another procedure for converting scaled differences. Students construct a 4x4 matrix for the distances between 4 cities and then construct a map from a given 4x4 matrix. Create a matrix that is impossible to solve because the distances are not sides of a triangle (such as multiples of 2, 3, and 7).

3. *LEARNER OUTCOME: Students demonstrate understanding of concepts related to SPACE and DIMENSIONALITY.*

Technology is introducing new concepts of dimensionality and space. Students will need to transfer their knowledge as these concepts change. Fractal geometry is an example of the new concepts of space that influence the lives of the students. The intervention teacher uses geometry explorations to increase secondary students' understanding of space and dimensionality. Students construct a fractal by repeatedly applying a procedure to each segment in the figure produced.

TITLE: Is the Area Shrinking and the Perimeter Enlarging?

OBJECTIVE: Students investigate area and perimeter.

MATERIALS: Diagram of an equilateral triangle with 8 inch sides. Scissors

PROCEDURES: The Sierpinski triangle is formed by drawing an equilateral triangle, reducing the triangle by a factor of 1/2, and making three paper copies of the reduced triangle. Reduced triangles are placed on the original triangle, one at each vertex, and the uncovered portion of the large triangle (marked with an x in the diagram below) is removed by blackening it on the paper. (This process is easily done on a computer in Logo.) Students discuss both the area and the perimeter of the removed and the remaining triangles. Students repeat the steps on each of the three remaining triangles and record the new areas and perimeters. They make conjectures about expressions for area and perimeter after n repetitions of the process.

EXTENSIONS: Students make conjectures based on starting with non-equilateral triangles. Students draw sketches of the resulting new figures. The computer output can be compared to the students' work.

4. *LEARNER OUTCOME: Students demonstrate understanding of MEASUREMENT concepts.*

Measurement activities are used in problem-solving situations where students explore their observations to develop formulas. Secondary students explore measurement concepts as they develop concepts in algebra and geometry. Graphics calculators and computer software are useful for investigations. Students use traditional measurement concepts in Euclidian geometry and in analytic geometry. New uses and means of measurement are being developed in our technological age, thus secondary students need an awareness of measurement from multiple perspectives. The intervention teacher chooses activities that enable students to integrate concepts from algebra and geometry as they develop their concepts of measurement.

TITLE: Angle Measure

OBJECTIVE: Students investigate properties of angles by making an angle measurement device.

MATERIALS: A mathematical compass and scissors for each student and a set of diagrams of acute, obtuse and right angles.

PROCEDURES: Students create their own devices for measuring angles. First they discuss angles in the classroom, such as the angle that the open door makes with the wall, and what makes an angle "big". They construct a circle with their compass, fold the circle in half (through the center) and cut on the fold line to produce two half-circles (180 degrees). Fold one half-circle in half and cut a quarter-circle (90 degrees). Fold and then cut one quarter-circle in half for an eighth of a circle (45 degrees). The second quarter-circle (90 degrees) is folded into thirds and then cut into a 60 degree and 30 degree angle. Students use their "wedges" to measure angles found in the classroom.

EXTENSIONS: Students discuss the advantages and disadvantages of assigning 90 degrees to a right angle (why not use 100 degrees for a right angle?). They experiment with adding and subtracting their angle measurement devices to measure the angles in the diagrams and angles in the classroom.

5. *LEARNER OUTCOME: Students demonstrate understanding of CHANGE concepts on patterns and functions.*

Everything in life is subject to change. Students develop understanding of change concepts on patterns and functions then apply this mathematical understanding to new situations. Secondary students use inductive reasoning to make conjectures from their observations. They employ deductive reasoning to prove their conjectures. Students develop their ability to apply the concepts of mathematical change to resolve problems they encounter in life.

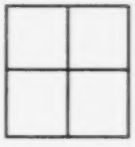
The intervention teacher designs activities that integrate ideas from algebra, geometry, and discrete mathematics to provide students with mathematical models to explore the concepts of change.

TITLE: Sum of Squares

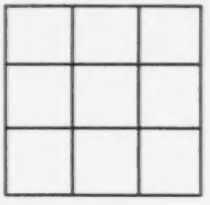
OBJECTIVE: Students investigate patterns.

MATERIALS: Toothpicks, grid paper

PROCEDURES: The pattern that emerges from the total number of smaller squares that are contained in larger squares requires students to explore measuring and quantifying change. Students will need manipulatives, such as toothpicks or straws. Students draw a square on grid paper and count the total number of squares contained within the square grid and record their observations. A 1x1 square contains 1 1x1 square. A 2x2 square contains 1 2x2 square and 4 1x1 squares. A 3x3 contains 1 3x3, 4 2x2, and 9 1x1 squares.



size of the large square	total number of squares in the square
1x1	1
2x2	5 (1 + 4)
3x3	14 (1 + 4 + 9)



EXTENSIONS: Students discover that the sum of squares concretely is the total sum of "squares" algebraically. Students generalize the pattern to a k x k square and test their conjectures, e.g.,

$1^2 + 2^2 + 3^2 + 4^2 + \ldots + k^2$

6. *LEARNER OUTCOME: Students demonstrate understanding of concepts related to MATHEMATICAL STRUCTURE.*

The basis of mathematical structure is the ability to relate concepts to one another in a problem-solving situation and then to connect these concepts to known procedures. Today's information age creates the need for students to understand discrete (non-continuous) structures, logic, and matrices. The concepts and methods of discrete mathematics, logic, and matrices are essential to solving problems using computer methods.

The intervention teacher provides activities for students that encourage them to expand their understanding of mathematical structures and create new uses for logical thinking and making assumptions. Writing about mathematics is an integral part of developing mathematical power.

TITLE: What's Wrong?

OBJECTIVE: Students analyze an incorrect algebraic procedure and explain the error in writing.

MATERIALS: Examples of incorrect algebraic procedures.

PROCEDURES: Students analyze a sample incorrect algebraic procedure to find the error and then explain to another student using algebra tiles the error and the correct algebraic procedure. For example,

$$\text{Is } 2(3x + 4) = 6x + 4 \text{ ?}$$

EXTENSIONS: Students work in pairs to predict incorrect algebraic procedures for a given problem, explaining the possible common errors.

7. *LEARNER OUTCOME: Students demonstrate understanding of DATA concepts related to both CERTAIN and UNCERTAIN events.*

Students live in an information age and thus need to process and use data effectively. Collecting, representing, and processing data through simulations and sampling activities enhance awareness of statistical experiments. The language of statistics is essential in communication. Topics of data concepts related to both certain and uncertain events (probability) are integrated with other mathematical concepts and connected with other academic subjects. The intervention teacher chooses activities that challenge students to explore real data.

TITLE: The Data Says ...

OBJECTIVE: Students investigate charting and interpreting data.

MATERIALS: Research materials with data relevant to national expenditures.

PROCEDURES: Students research and chart data on expenditures, such as for the military, social services, and education. They chart and graph their data. Then, in small groups, they prepare arguments based on their data. For example, one day they argue that the military are underfunded, the next day they argue that the military are overfunded. Discussions include methods of obtaining, presenting, and interpreting data.

EXTENSIONS: Students research and interpret data on the relationship between level of education and expected salaries.

SUMMARY OF THE INTERVENTION PROCESS IN MATHEMATICS

As mentioned earlier, the breadth of coverage of mathematics for the intervention student should span various TYPES, CONTEXTS, and CONCEPTS. TYPES of mathematical coverage should include writings, investigations, applications, non-routine problems, projects, and mathematical recreations. The CONTEXTS should vary among individual, cooperative group, and interdisciplinary while utilizing tools, manipulatives, calculators, computers, and other technology. Graphing calculators and computers are daily tools for the intervention student, not the exception. The seven core CONCEPTS are based upon the 54 NCTM Standards, namely; (1) number, (2) mathematical procedures, (3) space and dimensionality, (4) measurement, (5) change, (6) mathematical structures, and (7) data. The methodology must be developmentally-appropriate, implying a constructionist philosophy that expects the student to take on the role as an active mathematician and implies that the teacher take on the role as a facilitator. That is to say, the process, understanding and meaning of mathematics, is emphasized; the product, the correct answer and drill, is de-emphasized.

The examples listed in this document stress the need for high teacher expectations. However, comparing these examples to the required new state assessment instruments and mathematics portfolio entries shows them to be of similar quality and expectations. This mathematics intervention process addresses the world of tomorrow, a world where all students need to be mathematically empowered. The purpose of this mathematics intervention process, based on immersing students in meaningful mathematical activities, is to prepare our children for success.

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